

Implication of the observed $e^+e^- \rightarrow p\bar{p}\pi^0$ for studying the $p\bar{p} \rightarrow \psi(3770)\pi^0$ process

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We study the charmonium $p\bar{p} \rightarrow \psi(3770)\pi^0$ reaction using effective lagrangian approach where the contributions from well established N^* states are considered, and all parameters are fixed in the process of $e^+e^- \rightarrow p\bar{p}\pi^0$ at center of mass energy $\sqrt{s} = 3.773$ GeV. The experimental data on the line shape of the mass distribution of the $e^+e^- \rightarrow p\bar{p}\pi^0$ can be well reproduced. Based on the studying of $e^+e^- \rightarrow p\bar{p}\pi^0$, the total and differential cross sections of the $p\bar{p} \rightarrow \psi(3770)\pi^0$ reaction are predicted. At the same time we evaluated also the cross sections of the $p\bar{p} \rightarrow \psi(3686)\pi^0$ reaction. It is shown that the contribution of nucleon pole to this reaction is largest close to the reaction threshold. However, the interference between nucleon pole and the other nucleon resonance can still change the angle distributions significantly. Those theoretical results may be test by the future experiments at PANDA.

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I. INTRODUCTION

As a forthcoming facility in future, the Anti-Proton Annihilations at Darmstadt (PANDA) experiment will focus on the production of charmonium, which is govern by nonperturbative effect of quantum chromodynamics (QCD) [1]. Before PANDA run, there were pioneering theoretical studies of the charmonium production in the $p\bar{p}$ annihilation processes [2–9]. By calculating two hadron-level diagrams introduced by the Born approximation, Gaillard and Maiani firstly studied the differential cross section of the charmonium production plus a soft pion in the $p\bar{p}$ reaction [2]. In Ref. [3], the cross sections of the charmonium (Ψ) production accompanied by a light meson (m) from the process of $p\bar{p} \rightarrow \Psi + m$ was calculated by combing with the measured partial decay widths of charmonium decay into $p\bar{p}m$. And then, Barnes and Li proposed an initial state light meson emission model for the near threshold associated charmonium production processes $p\bar{p} \rightarrow \pi^0\Psi$ ($\Psi = \eta_c, J/\psi, \psi', \chi_{c0}, \chi_{c1}$), and the total and differential cross sections for these reactions were evaluated [4–6]. It is also found that the cross section of $p\bar{p} \rightarrow \pi^0\Psi$ near threshold may be affected by the Pauli $J/\psi p\bar{p}$ coupling [5]. Furthermore, Lin, Xu and Liu revisited the issue of the production of charmonium plus a light meson at PANDA, where the contribution of form factors (FFs) to these processes are included [7]. Recently, Pire *et al.* studied the associated production of a J/ψ and a pion in antiproton-nucleon annihilation in the framework of QCD collinear factorization [8], while in Ref. [9], the exclusive charmonium production process $p\bar{p} \rightarrow \pi^0 J/\psi$ was studied within a nucleon-pole exchange model by including off-shell hadronic FFs and a complete Lorentz structure with a $\bar{p}pJ/\psi$ Pauli strong coupling. The

contributions from the intermediate N^* states are also studied in Ref. [9], and it was found that one can not ignore the contributions of the N^* resonances in the $p\bar{p} \rightarrow \pi^0 J/\psi$ reaction.

The experimental activity on the charmonium decays have run in parallel. These decays are of interest because they can be used to study the associated charmonium production in $p\bar{p}$ annihilation. In 2014, the BESIII Collaboration reported the analysis of $e^+e^- \rightarrow p\bar{p}\pi^0$ in the vicinity of $\psi(3770)$ [10]. In addition to the Born cross section of $e^+e^- \rightarrow p\bar{p}\pi^0$, the corresponding $p\pi^0$ and $\bar{p}\pi^0$ invariant mass distributions of $e^+e^- \rightarrow p\bar{p}\pi^0$ process are also measured [10]. These new experimental information in Ref. [10] allows us to further perform a comprehensive study of $e^+e^- \rightarrow \psi(3770) \rightarrow p\bar{p}\pi^0$, which stimulates our interest to study the contribution of excited nucleon resonances (N^*) to $e^+e^- \rightarrow \psi(3770) \rightarrow p\bar{p}\pi^0$ and $\psi(3770)$ production from $p\bar{p} \rightarrow \psi(3770)\pi^0$ reaction.

The nucleon is the simplest system in which the three colors of QCD can combine to form a colorless object, thus it is important to understand the internal quark-gluon structure of the nucleon and its excited N^* states, and the study of excited N^* states is an interested research field of hadron physics [11], which can make our knowledge of hadron spectrum abundant. A very important source of information for the nucleon internal structure is the N^* mass spectrum as well as its various production and decay rates, while the charmonium decay into $p\bar{p}\pi^0$ is an ideal platform to study excited N^* nucleon resonances, because it provides an effective isospin 1/2 filter for the πN system due to isospin conservation [12–14].

In this work, we introduce excited N^* nucleon resonances in the process of $e^+e^- \rightarrow \psi(3770) \rightarrow p\bar{p}\pi^0$. By fitting the $p\pi^0$ and $\bar{p}\pi^0$ invariant mass distributions of the cross section of $e^+e^- \rightarrow p\bar{p}\pi^0$, we extract the information of couplings of $N^*N\pi$ and $\psi(3770)N^*\bar{N}$, which not only reflects the inner features of discussed N^* , but also helps us to learn the role played by N^* in the $e^+e^- \rightarrow \psi(3770) \rightarrow p\bar{p}\pi^0$.

Based on our studying on the $e^+e^- \rightarrow \psi(3770) \rightarrow p\bar{p}\pi^0$ process, we move forward to study the $p\bar{p} \rightarrow \psi(3770)\pi^0$ reaction, which is due to the cross relation between the $\psi(3770) \rightarrow p\bar{p}\pi^0$ decay and the $p\bar{p} \rightarrow \psi(3770)\pi^0$ reac-

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tion [13]. Here, these extracted parameters from our study of $e^+e^- \rightarrow \psi(3770) \rightarrow p\bar{p}\pi^0$ will be employed to estimate the production rate of $p\bar{p} \rightarrow \psi(3770)\pi^0$ and relevant features. We calculate the total and differential cross sections of the $p\bar{p} \rightarrow \psi(3770)\pi^0$ reaction. It is shown that the contribution of nucleon pole to this reaction is the largest close to the reaction threshold. However, the interference between nucleon pole and the other nucleon resonance affects significantly and could change the angle distributions clearly. Additionally, there were abundant experimental data of $\psi(3686) \rightarrow p\bar{p}\pi^0$ given by BESIII [14], where BESIII released the branching ratio $B(\psi(3686) \rightarrow p\bar{p}\pi^0) = (1.65 \pm 0.03 \pm 0.15) \times 10^{-4}$ and the measured $p\pi^0$ and $\bar{p}\pi^0$ invariant mass spectra [14]. This experimental status related to $\psi(3686)$ makes us extend the above study to the $\psi(3686) \rightarrow p\bar{p}\pi^0$ decay, and also the $p\bar{p} \rightarrow \psi(3686)\pi^0$ reaction. Our studies provide valuable information to future experimental exploration of $\psi(3770)$ and $\psi(3686)$ productions plus a pion through the $p\bar{p}$ interaction at PANDA.

This paper is organized as follows. After introduction in Sec. I, we present the detailed study of $e^+e^- \rightarrow p\bar{p}\pi^0$ by including the excited N^* nucleon resonances (see Sec. II). In Sec. III, we further calculate $p\bar{p} \rightarrow \psi(3770)\pi^0$ by combining with these results obtained in Sec. II. In Sec. IV, we adopt the similar approach to study $\psi(3686) \rightarrow p\bar{p}\pi^0$ decay and the $p\bar{p} \rightarrow \psi(3686)\pi^0$ process. The paper ends with a discussion and conclusion.

II. EXCITED N^* NUCLEON RESONANCE CONTRIBUTIONS TO $e^+e^- \rightarrow \psi(3770) \rightarrow p\bar{p}\pi^0$

First, we study the process $e^+e^- \rightarrow p\bar{p}\pi^0$ with an effective Lagrangian approach. In hadron level, the process $e^+e^- \rightarrow p\bar{p}\pi^0$ in the vicinity of $\psi(3770)$ is described by the diagrams shown in Fig. 1. In Fig. 1 (a), e^+ and e^- annihilate into photon, which couples with charmonium $\psi(3770)$. And then, $\psi(3770)$ interacts with final states, where we consider the contributions from nucleon-pole ($\equiv P_{11}$) with $J^P = \frac{1}{2}^+$ and five N^* states that are well established [15]: $N(1440)$ ($\equiv P_{11}$) with $J^P = \frac{1}{2}^+$, $N(1520)$ ($\equiv D_{13}$) with $J^P = \frac{3}{2}^-$, $N(1535)$ ($\equiv S_{11}$) with $J^P = \frac{1}{2}^-$, $N(1650)$ ($\equiv S_{11}$) with $J^P = \frac{1}{2}^-$, and $N(1720)$ ($\equiv P_{13}$) with $J^P = \frac{3}{2}^+$. Additionally, we also consider the background contribution, where the e^+e^- annihilation directly into $p\bar{p}\pi$ without intermediate $\psi(3770)$, which is shown in Fig. 1 (b).

To compute the contributions of these terms, we use the effective interaction Lagrangian densities for each vertex. For the $\gamma\psi(3770)$ coupling, we adopt the vector meson dominant (VMD) model, where a vector meson couples to a photon is described by [16]

$$\mathcal{L}_{V\gamma} = -\frac{eM_V^2}{f_V} V_\mu A^\mu. \quad (1)$$

In above expression, M_V and f_V are the mass and the decay constant of the vector meson, respectively. The decay constant

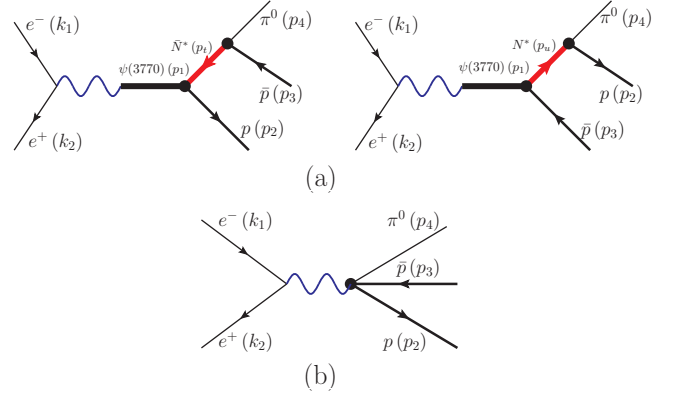


FIG. 1: (color online). The Feynman diagrams for the process $e^+e^- \rightarrow p\bar{p}\pi^0$ in the vicinity of $\psi(3770)$.

e/f_V can be fitted through $V \rightarrow e^+e^-$:

$$e/f_V = \left[\frac{3\Gamma_{V \rightarrow e^+e^-} M_V^2}{8\alpha |\vec{p}|^3} \right]^{1/2} \simeq \left[\frac{3\Gamma_{V \rightarrow e^+e^-}}{\alpha M_V} \right]^{1/2}, \quad (2)$$

where $|\vec{p}| = (M_V^2 - 4m_e^2)^{1/2}/2 \simeq M_V/2$ is three momentum of an electron in the rest frame of the vector meson. $\alpha = e^2/(4\pi) = 1/137$. Using $B(\psi(3770) \rightarrow e^+e^-) = (9.6 \pm 0.7) \times 10^{-6}$ [15], we obtain $e/f_{\psi(3770)} = 0.0053$.

The $J/\psi N \bar{N}$ and $NN\pi$ couplings are described by:

$$\mathcal{L}_{\pi NN} = -\frac{g_{\pi NN}}{2m_N} \bar{N} \gamma_5 \gamma_\mu \tau \cdot \partial^\mu \pi N, \quad (3)$$

$$\mathcal{L}_{\psi NN} = -g_{\psi NN} \bar{N} \gamma_\mu V^\mu N, \quad (4)$$

where V^μ stands for the vector field of $\psi(3770)$. We take $g_{\pi NN} = 13.45$.

For the $N^* N \pi$ and $\psi N^* \bar{N}$ vertexes, we adopt the Lagrangian densities as used in Refs. [17–23]:

$$\mathcal{L}_{\pi NP_{11}} = -\frac{g_{\pi NP_{11}}}{2m_N} \bar{N} \gamma_5 \gamma_\mu \tau \cdot \partial^\mu \pi R_{P_{11}} + h.c., \quad (5)$$

$$\mathcal{L}_{\pi NS_{11}} = -g_{\pi NS_{11}} \bar{N} \tau \cdot \pi R_{S_{11}} + h.c., \quad (6)$$

$$\mathcal{L}_{\pi NP_{13}} = -\frac{g_{\pi NP_{13}}}{m_N} \bar{N} \tau \cdot \partial_\mu \pi R_{P_{13}}^\mu + h.c., \quad (7)$$

$$\mathcal{L}_{\pi ND_{13}} = -\frac{g_{\pi ND_{13}}}{m_N^2} \bar{N} \gamma_5 \gamma^\mu \tau \cdot \partial_\mu \partial_\nu \pi R_{D_{13}}^\nu + h.c., \quad (8)$$

$$\mathcal{L}_{\psi NP_{11}} = -g_{\psi NP_{11}} \bar{N} \gamma_\mu V^\mu R_{P_{11}} + h.c., \quad (9)$$

$$\mathcal{L}_{\psi NS_{11}} = -g_{\psi NS_{11}} \bar{N} \gamma_5 \gamma_\mu V^\mu R_{S_{11}} + h.c., \quad (10)$$

$$\mathcal{L}_{\psi NP_{13}} = -ig_{\psi NP_{13}} \bar{N} \gamma_5 V_\mu R_{P_{13}}^\mu + h.c., \quad (11)$$

$$\mathcal{L}_{\psi ND_{13}} = -g_{\psi ND_{13}} \bar{N} V_\mu R_{D_{13}}^\mu + h.c., \quad (12)$$

where R is a N^* field.

For the intermediate nucleon-pole or N^* state, a Breit-Wigner form of its propagator $G_J(q)$ can be written as [24]

$$G_{\frac{1}{2}}(q) = i \frac{q \not{+} M_{N^*}}{q^2 - M_{N^*}^2 + iM_{N^*} \Gamma_{N^*}} \quad (13)$$

for $J = \frac{1}{2}$, and

$$G_{\frac{3}{2}}^{\mu\nu}(q) = i \frac{q + M_{N^*}}{q^2 - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*}} \left(-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3m}(\gamma_\mu q_\nu - \gamma_\nu q_\mu) + \frac{2}{3} \frac{q_\mu q_\nu}{q^2} \right) \quad (14)$$

for $J = \frac{3}{2}$. In Eqs. (13) and (14), M_{N^*} and Γ_{N^*} are the masses and widths of these intermediate N^* states, respectively. The values used in the present work for M_{N^*} and Γ_{N^*} are summarized in Table I.

TABLE I: Relevant resonant parameters for N^* states. The values are taken from Particle Data Book [15].

N^*	M_{N^*} (MeV)	Γ_{N^*} (MeV)
$N(938)$	938	0
$N(1440)$	1430	350
$N(1520)$	1515	115
$N(1535)$	1535	150
$N(1650)$	1655	140
$N(1720)$	1720	250

On the other hand, we also need to introduce the form factors for these intermediate off-shell N^* (N), which are taken as in Refs. [25–28]:

$$F(q^2) = \frac{\Lambda^4}{\Lambda^4 + (q^2 - M_{N^*}^2)^2}, \quad (15)$$

where the cutoff parameter Λ can be parameterized as

$$\Lambda = M_{N^*} + \beta\Lambda_{QCD}, \quad (16)$$

with $\Lambda_{QCD} = 220$ MeV. The parameter β will be determined by fitting the experimental data.

For the background contribution depicted in Fig. 1 (b), we construct the amplitude in analogy of Ref. [29]:

$$\mathcal{M}_{NoR} = g_{NoR}\bar{v}(k_2)e\gamma^\mu u(k_1)\frac{1}{s}\bar{u}(p_2)\gamma^\mu\gamma_5 v(p_3)\mathcal{F}_{NoR}(s), \quad (17)$$

with $\mathcal{F}_{NoR}(s) = \exp(-a(\sqrt{s} - \sum_f m_f)^2)$, where $\sum_f m_f$ means the mass of the final states are summed over. The parameter a will be fitted to the experimental measurements, and s is the invariant mass square of the e^+e^- system.

In the phenomenological Lagrangian approaches, the relative phases between amplitudes from different diagrams are not fixed. Generally, we should introduce a relative phase between different amplitudes as free parameters, and the total

amplitude can be written as:

$$\begin{aligned} \mathcal{M}_{e^+e^- \rightarrow p\bar{p}\pi^0} &= \mathcal{M}_{NoR}e^{i\phi_{NoR}} + \bar{v}(k_2)e\gamma^\mu u(k_1)\frac{-g^{\mu\nu}}{s}em_\psi^2/f_\psi \\ &\times \frac{-g_{\nu\alpha} + \frac{p_{\psi\nu}p_{\psi\alpha}}{m_\psi^2}}{s - m_\psi^2 + im_\psi\Gamma_\psi} \left(\mathcal{M}_N^\alpha + \sum_{N^*} \mathcal{M}_{N^*}^\alpha e^{i\phi_{N^*}} \right), \end{aligned} \quad (18)$$

where $\mathcal{M}_{N^*(N)}^\alpha$ describing the subprocesses $\psi(3770) \rightarrow p\bar{p}\pi^0$ are given completely in appendix.

The differential cross section is given by [30]

$$d\sigma_{e^+e^- \rightarrow p\bar{p}\pi^0} = \frac{(2\pi)^4 \sum |\mathcal{M}_{e^+e^- \rightarrow p\bar{p}\pi^0}|^2}{4\sqrt{(k_1 \cdot k_2)^2}} d\Phi_3, \quad (19)$$

and the phase space factor is given by

$$d\Phi_3 = \frac{1}{(2\pi)^9} \frac{1}{8\sqrt{s}} |\vec{p}_3^*||\vec{p}_2^*| d\Omega_3^* d\Omega_2 dm_{p\pi}, \quad (20)$$

with $\sum |\mathcal{M}|^2$ averaging over the spins of the initial e^+e^- and summing over the polarizations of the final states $p\bar{p}$.

As we can see in the appendix, in the tree-level approximation, only the products like $g_{N^*} \equiv g_{VNN^*}g_{\pi NN^*}$ enter in the invariant amplitudes. They are determined with the use of MINUIT, by fitting to the low energy experimental data on mass distribution of $e^+e^- \rightarrow p\bar{p}\pi^0$ at $\sqrt{s} = 3.773$ GeV [10]. So far we have fifteen unknown parameters: six g_{N^*} , six phase angles ϕ_{N^*} and ϕ_{NoR} , one cutoff β in the form factors and two parameters g_{NoR} and a in direct production amplitude Eq. (17). We perform those fifteen-parameter χ^2 fits to the BESIII experiment data on the invariant mass distribution at 3.773 GeV below 1.8 GeV, and make use of the total cross section information in Ref. [10]. Here, we do not consider the invariant mass region beyond 1.8 GeV, where contains large contribution from higher mass N^* states and other complicated resonance which decays to $p\bar{p}$. In Ref. [9], it was pointed that in the case of $\bar{p}p \rightarrow \pi^0 J/\psi$ reaction the higher mass N^* resonances are needed. Indeed, in the present case, if we go beyond 1.8 GeV, we need also the higher mass N^* states. On the other hand, we did also another calculation including the contributions of higher spin nuclear excited states, $N(1675)5/2^-$ and $N(1680)5/2^+$. It is find that their contributions are quite small and the fitted parameters for the other nuclear resonance are little changed. Thus, we will not include the contributions of this two states in this work.

We get a minimal $\chi^2/dof = 1.03$ with the fitted cut-off parameter $\beta = 6.2 \pm 3.5$. The parameters appearing in direct amplitude Eq. (17) are $g_{NoR} = 0.45 \pm 0.02$, $\phi_{NoR} = 4.84 \pm 0.20$ Rad and $a = 0.84 \pm 0.02$. The other fitted parameters are compiled in Table II. The fitted results are shown in Fig. 2 compared with the experimental data taken from Ref. [10], where the green dashed line stands for the background contribution, the orange dotted line stands for the nucleon-pole contribution, the red line is the full result, and other lines show the contributions from different N^* resonances. Notice that we have converted the experimental event to physical differential cross section using the experimental value $\sigma_{total} = 7.71$ pb at 3.773 GeV

[10]. Our results can describe the two clear peaks around 1.5 GeV and 1.7 GeV, thanks to the contributions from $N(1520)$, $N(1535)$ and $N(1650)$ resonances. The contribution from the nucleon pole is small, while the background contribution is quite large.

In Fig. 2, it is interesting to see large interfering effects between different contributions. At the low $M_{\bar{p}\pi}$ region around 1.1 – 1.3 GeV, large cancelation between the nucleon pole and the background leads to quiet suppressed spectrum, and the bump structure from the nucleon pole just disappears. From the two-peak region around 1.4 – 1.8 GeV, we can directly see that, the background contribution plus N^* contribution (means without interfering contribution) is not able to reach the data peak, it indicates a large enhancement between the background contribution and N^* contribution thanks to the interfering effect.

TABLE II: The fitted parameters in the process $e^+e^- \rightarrow p\bar{p}\pi^0$, where $g_{N^*} = g_{\psi(3770)NN^*}g_{\pi NN^*}$. For nucleon, g_N is defined as $g_N = g_{\psi(3770)NN}g_{\pi NN}$

N^*	$g_{N^*} (\times 10^{-3})$	$\phi_{N^*} (\text{rad})$
$N(938)$	8.00 ± 0.46	—
$N(1440)$	1.92 ± 0.98	6.09 ± 0.38
$N(1520)$	0.28 ± 0.24	3.74 ± 1.07
$N(1535)$	1.74 ± 1.34	2.99 ± 0.67
$N(1650)$	1.99 ± 0.18	2.17 ± 0.19
$N(1720)$	1.14 ± 0.63	6.02 ± 0.71

III. THE $\psi(3770)$ PRODUCTION IN THE PROCESS $p\bar{p} \rightarrow \psi(3770)\pi^0$

A charmonium plus a light meson π can be produced by the low energy $p\bar{p}$ annihilation process. The tree level diagrams for the $p\bar{p} \rightarrow \psi(3770)\pi^0$ reaction are depicted in Fig. 3. It is worth to mention that the effect of the N^* resonances in the cross channel of Fig. 3 has been studied firstly in the $\bar{p}p \rightarrow \pi^0 J/\psi$ reaction [9]. It was found that the contributions from the N^* resonances in the $\bar{p}p \rightarrow \pi^0 J/\psi$ reaction are important. In the present work, we extend the model of Ref. [9] to the process of the higher charmonium states [$\psi(3770)$ and $\psi(3686)$] production.¹

The differential cross section of the $p\bar{p} \rightarrow \pi^0\psi(3770)$ reac-

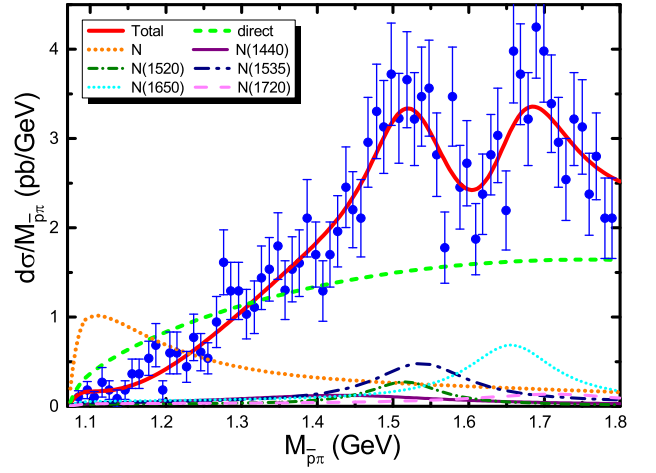


FIG. 2: (color online). The fitted mass spectrum of the process $e^+e^- \rightarrow p\bar{p}\pi^0$ at $\sqrt{s} = 3.773$ GeV comparing to the experiment data. The experiment data are taken from Ref. [10]. The green dashed line stands for the background contribution, the orange dotted line stands for the nucleon-pole contribution, the red line is the full result, and other lines show the contributions from different N^* resonances. Notice that the experimental event is converted to physical differential cross section using the experimental total cross section at $\sqrt{s} = 3.773$ GeV [10].

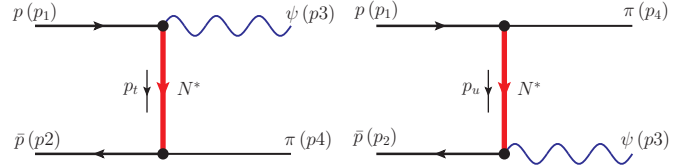


FIG. 3: (color online). The typical Feynman diagrams for the process $p\bar{p} \rightarrow \pi^0\psi(3770)$.

tion at center of mass (c.m.) frame can be expressed as [15]

$$\frac{d\sigma_{p\bar{p} \rightarrow \pi^0\psi(3770)}}{d\cos\theta} = \frac{1}{32\pi s} \frac{|\vec{p}_3^{\text{cm}}|}{|\vec{p}_1^{\text{cm}}|} \sum |\overline{\mathcal{M}}|^2, \quad (21)$$

where θ denotes the angle of the outgoing π^0 relative to beam direction in the c.m. frame, \vec{p}_1^{cm} and \vec{p}_3^{cm} are the three-momentum of the proton and $\psi(3770)$ in c.m. frame, respectively, while the total invariant scattering amplitude \mathcal{M} is given in appendix using cross symmetry.

With the parameters determined from the process of $e^+e^- \rightarrow p\bar{p}\pi^0$, we calculate the total and differential cross sections of $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction. In Fig. 4, we show our results for the total cross section of the $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction as a function of the invariant mass (E_{cm}) of $\bar{p}p$ system. At $E_{\text{cm}} = 5.26$ GeV, the total cross section is 0.056 nb, and it is under the upper limit of the value obtained in Ref. [10].

From Fig. 4, we see that the nucleon pole gives largest contribution, and becomes dominant in the region $E_{\text{cm}} > 5.0$ GeV. This is because in the reaction of $p\bar{p} \rightarrow \psi(3770)\pi^0$, the four momentum square, q^2 , of nucleon or other nucleon resonance is smaller than 0, and the propagator $\frac{1}{q^2 - M^2}$ will in-

¹ We mention that the Regge exchange may be important, unfortunately, the information of the Regge propagators are scarce and we hope we can include the Regge contribution in the future.

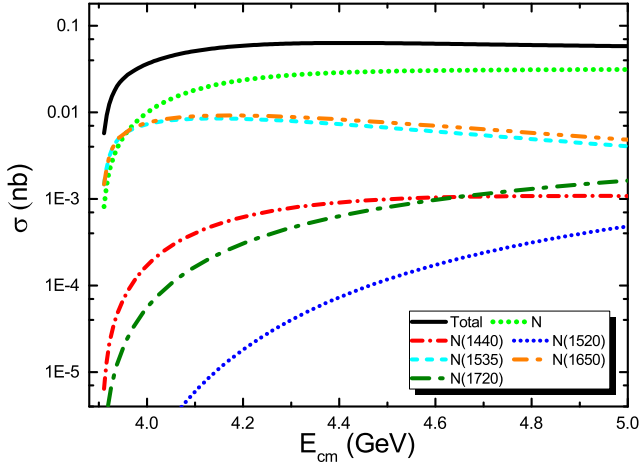


FIG. 4: (color online). Total cross section of the $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction. The black line is total result, and other lines show the contributions from different N^* resonances.

crease the contribution of nucleon because of its small mass. Besides, it is found that the contributions from N^* state with different quantum numbers have quite different behavior. The contributions from $N(1535)$ and $N(1650)$ with $J^P = \frac{1}{2}^-$ decrease at E_{cm} around 4.2 GeV, while the others increase all the time. Overall, the total cross section become quiet flat while $E_{\text{cm}} > 4.1$ GeV.

In addition, we also calculate the angular distribution of the $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction at $E_{\text{cm}} = 4.0, 4.25, 4.5, 4.75$ and 5.0 GeV. The numerical results are shown in Fig. 5. We can see that there emerges an obvious peak at the backward angles (around $\cos\theta \sim -0.8$) at $E_{\text{cm}} \geq 4.25$ GeV produced by the contributions of nucleon results in the u -channel, while the larger results at the forward angles is due to the t -channel nucleon resonances contributions.

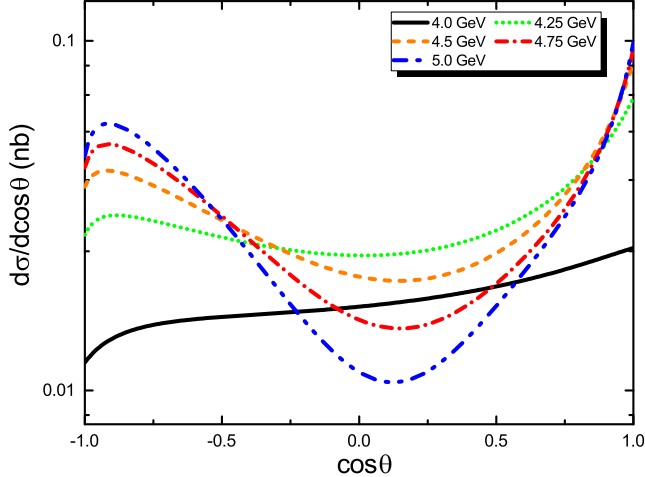


FIG. 5: (color online). Angular distributions of the $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction with full contribution.

In Fig. 6, we show the numerical results of the angular distributions by only considering the contribution from the nu-

cleon pole. We can see that the angular distributions are symmetry between the backward and forward angles. Comparing Fig. 5 with Fig. 6, we see that, there is a big difference between the full contribution and the only nucleon contribution. Our model predictions may be tested by the future experiments.

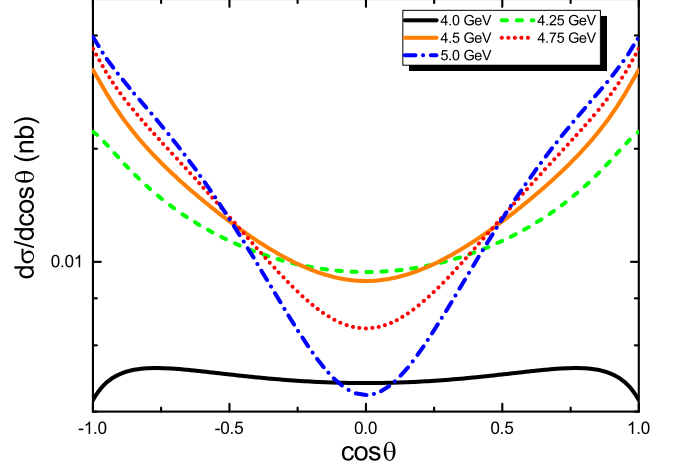


FIG. 6: (color online). Angular distributions of the $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction considered only the contribution from the nucleon pole.

Note that the exchanged nuclear resonances in Fig. 3 are far off mass shell, and the form factors for exchanged nuclear resonances here should be different with those that have been used for the $e^-e^+ \rightarrow \psi(3770) \rightarrow p\bar{p}\pi^0$ reaction. We know that the form factors can be directly related to the hadron structure. However, the question of hadron structure is still very open, we have to adjust the form factor to fit the experimental data, and the hadronic form factors are commonly used phenomenologically [25–28]. The effects of these form factors could substantially change the predicted cross sections. Because of the lack of the available experimental measurements, we can not determine the form factors without ambiguities. In the present work, we take the same form factors for both $\bar{p}p \rightarrow \psi(3770)\pi^0$ reaction and $e^+e^- \rightarrow \psi(3770) \rightarrow \bar{p}p\pi^0$ reaction.

IV. THE IMPLICATION FOR $\psi(3686) \rightarrow p\bar{p}\pi^0$ AND $p\bar{p} \rightarrow \psi(3686)\pi^0$

For the process $\psi(3686) \rightarrow p\bar{p}\pi^0$, we first determine the coupling constant $g_{\psi(3686)NN}$, i.e., by using the Lagrangian in Eq. (4), $g_{\psi(3686)NN}$ can be fitted through the process $\psi(3686) \rightarrow p\bar{p}$. With the experimental value [15] $B(\psi(3686) \rightarrow p\bar{p}) = 2.8 \times 10^{-4}$, $g_{\psi(3686)NN}$ is determined to be

$$g_{\psi(3686)NN} = 9.4 \times 10^{-4}, \quad (22)$$

which is consistent with that given in Ref. [4].

In Ref. [14], BESIII released the $p\pi$ invariant mass spectrum of the process $\psi(3686) \rightarrow p\bar{p}\pi^0$ and decay width $\Gamma(\psi(3686) \rightarrow p\bar{p}\pi^0) = (1.65 \pm 0.03 \pm 0.15) \times 10^{-5}$. Similar to the case of $\psi(3770)$, we fit five coupling constants g_{N^*} ,

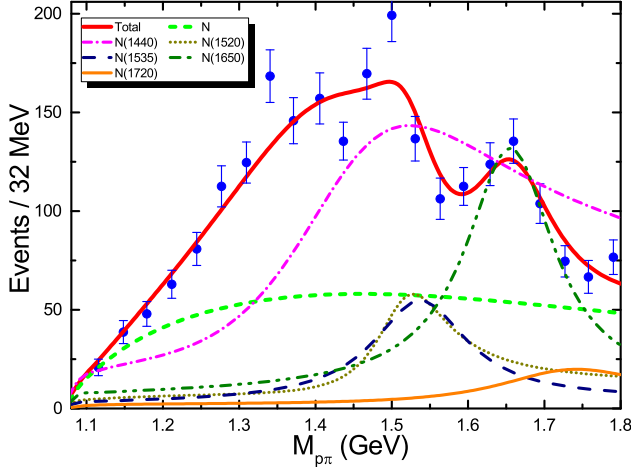


FIG. 7: (color online). The fitted $p\pi$ invariant mass spectrum of the process of $\psi(3686) \rightarrow p\bar{p}\pi^0$. The dashed green curve stands for the contribution of the nucleon pole, the solid red line stands for the full contributions, and other lines show the contributions from different N^* resonances. The experiment data are taken from Ref. [14].

five phase angles and a cut off parameter β to the experimental data. The fitted results are shown in Fig. 7. Here, one gets $\chi^2/d.o.f = 2.90$ and $\beta = 3.28 \pm 2.23$, while the fitted coupling constants g_{N^*} and phase angles are listed in Table III.

TABLE III: Fitted coupling constants g_{N^*} and phase angles ϕ_{N^*} in the process $\psi(3686) \rightarrow p\bar{p}\pi^0$, where $g_{N^*} = g_{\psi(3686)NN^*}g_{\pi NN^*}$

N^*	$g_{N^*} (\times 10^{-3})$	$\phi_{N^*} (\text{rad})$
$N(1440)$	5.10 ± 0.86	3.40 ± 0.22
$N(1520)$	2.27 ± 0.39	4.96 ± 1.10
$N(1535)$	0.51 ± 0.36	0.75 ± 0.64
$N(1650)$	0.76 ± 0.19	5.35 ± 0.92
$N(1720)$	0.98 ± 0.42	1.77 ± 0.99

In Fig. 7, the dashed curve stands for the contribution of the nucleon pole, the solid line stands for the full contributions, and other lines show the contributions from different N^* resonances. We see that we can describe the experimental data fairly well. Furthermore, we find that the peak between 1.6 GeV and 1.7 GeV mainly comes from the contribution of $N(1650)$.

There also exist quiet obvious interfering effects between different N^* contributions in Fig. 7. Close to $M_{p\pi} = 1.6$ GeV, comparing the $N(1440)$ contribution to the total contribution, one can see the $N(1440)$ contribution is "digged out" a valley by other N^* contributions. In the region of $M_{p\pi} > 1.7$ GeV, the total contribution is smaller than the $N(1440)$ contribution, i.e., the total contribution is suppressed by interfering terms. So, from Fig. 2 and Fig. 7, one can see how important the interference effect is. We will not be able to get a good fit

without interfering terms and arbitrary phase angles.

Additionally, we also calculated the branch fractions of $\psi(3686) \rightarrow (N^*\bar{p} + c.c.) \rightarrow p\bar{p}\pi^0$ from individual intermediate N^* (or p) state, with the fitted coupling constants listed in the Table III. Our results are shown in Table IV. The errors of our theoretical results are obtained from the errors of those fitted coupling constants of g_{N^*} . We also notice that in Ref. [14] BESIII also extracted the corresponding branching fractions without considering the interference of different intermediate N^* (or p) states, which is different from the treatment in the present work. Thus, in Table IV we further compare our result with the experimental results [14], we see that our results are in agreement within errors with that given in Ref. [14].

TABLE IV: The calculated branching fractions $\psi(3686) \rightarrow p\bar{p}\pi^0$ if considering individual intermediate N^* (or N) contribution, and the comparison with the experiment values of Ref. [14]. Here, all values are in the unit of 10^{-5} .

	Our results	The results in Ref. [14]
N	7.5	$6.42^{+0.20+1.78}_{-0.20-1.28}$
$N(1440)$	14 ± 4.5	$3.58^{+0.25+1.59}_{-0.25-0.84}$
$N(1520)$	2.8 ± 0.8	$0.64^{+0.05+0.22}_{-0.05-0.17}$
$N(1535)$	2.1 ± 3.0	$2.47^{+0.28+0.99}_{-0.28-0.97}$
$N(1650)$	4.9 ± 2.5	$3.76^{+0.28+1.37}_{-0.28-1.66}$
$N(1720)$	1.3 ± 1.0	$1.79^{+0.10+0.24}_{-0.10-0.71}$

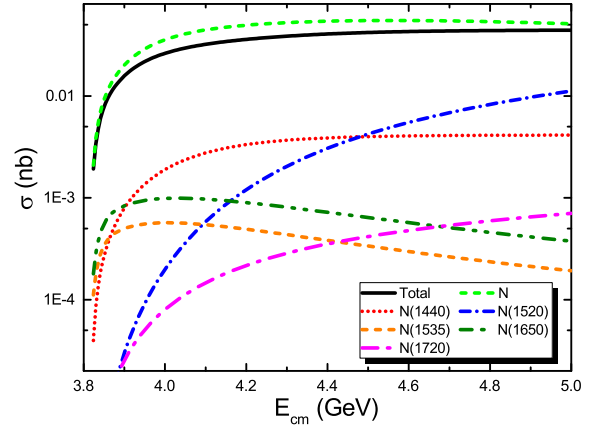


FIG. 8: (color online). The cross section of the process $p\bar{p} \rightarrow \pi^0\psi(3686)$. The black line is total result, and other lines shows the N^* contribution.

With these fitted parameters, we calculate the cross section of the process $p\bar{p} \rightarrow \pi^0\psi(3686)$ with cross symmetry. The results are shown in Fig. 8. One can see that the nucleon pole contribution is predominant in the whole energy region, while the contributions from other N^* states are small. In the higher energy region, the nucleon pole contribution is starting to decrease, while the full contribution increases slowly,

this behavior resembles the process $p\bar{p} \rightarrow \pi^0\psi(3770)$. Furthermore, it is noticed that, the discrepancy between the total result and the nucleon contribution is smaller than the case of $p\bar{p} \rightarrow \pi^0\psi(3770)$.

Finally, we show the angular distributions of the process $p\bar{p} \rightarrow \pi^0\psi(3686)$ in Figs. 9 and 10. Similar to Fig. 5, there is a peak in backward angle and a valley close to $\cos\theta = 0$. Comparing to the angular distribution with the nucleon contribution in Fig. 10, there exists obvious difference, since the nucleon contribution only is symmetry while total contribution is asymmetry.

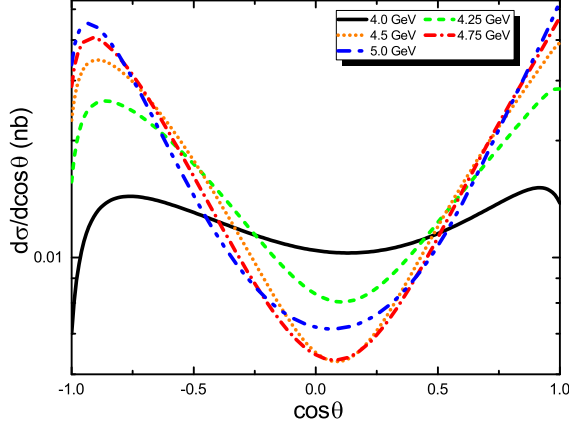


FIG. 9: (color online). the angular distribution of the process $p\bar{p} \rightarrow \pi^0\psi(3686)$. Each line shows the different c.m. energy.

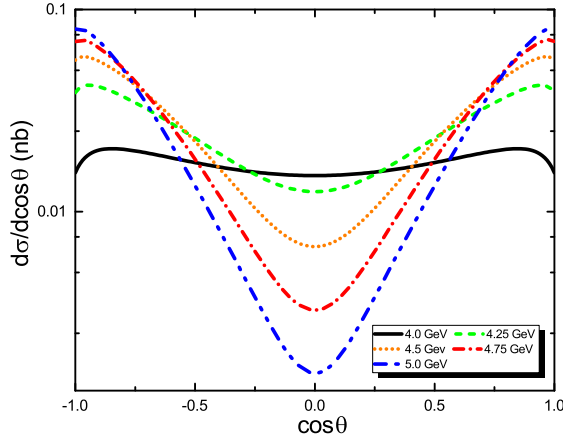


FIG. 10: (color online). the angular distribution of the process $p\bar{p} \rightarrow \pi^0\psi(3686)$ only considering the nucleon contribution. Each line shows the different c.m. energy.

V. DISCUSSION AND CONCLUSION

We have studied the $e^+e^- \rightarrow p\bar{p}\pi^0$ at 3.773 GeV c.m. energy and $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction within an effective la-

grangian approach. The $e^+e^- \rightarrow p\bar{p}\pi^0$ process is a good platform to study excited N^* nucleon resonances. We consider contributions from nucleon pole and five well established N^* states. First, we perform a χ^2 -fit to the experimental data on the mass distribution of the $e^+e^- \rightarrow p\bar{p}\pi^0$, from where we obtain the couplings of $\psi(3770)$ to these N^* states. It is shown that we can describe the experimental data quite well. In particular, the two bumps around 1.5 and 1.7 GeV can be well reproduced. We also find that the contribution of the nucleon pole is small comparing to the background contribution, and there exists large cancellation in low $M_{p\bar{p}}$ region.

Second, based on our results of the $e^+e^- \rightarrow p\bar{p}\pi^0$, we study the $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction with cross symmetry. We evaluate the total and differential cross sections of the $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction. The nucleon pole gives largest contribution to the $p\bar{p} \rightarrow \pi^0\psi(3770)$ reaction close to threshold. However, the interference terms between nucleon pole and the other nucleon resonance affects significantly and could change the angle distributions clearly. Our studies provide valuable information to future experimental exploration the $\psi(3770)\pi^0$ production through the $p\bar{p}$ interaction.

Additionally, we also study the $\psi(3686)$ production through the process $p\bar{p} \rightarrow \pi^0\psi(3686)$. Similarly to the case of $e^+e^- \rightarrow \psi(3770) \rightarrow p\bar{p}\pi^0$, we study firstly the decay process of $\psi(3686) \rightarrow p\bar{p}\pi^0$ to extract the parameters we needed. Then we study the $p\bar{p} \rightarrow \pi^0\psi(3686)$ reaction. We find that the contribution from the nucleon pole is dominant, while the angular distributions show a quite discrepancy induced by the N^* states.

We hope and expect that future experiments at PANDA will provide a test to our model and give more constraints on our theoretical study.

Acknowledgments

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Appendix: Scattering amplitudes of the subprocess

$\psi(3770) \rightarrow p\bar{p}\pi^0$ and the process $p\bar{p} \rightarrow \pi^0\psi(3770)$.

The tree level diagrams of the subprocess $\psi(3770) \rightarrow p\bar{p}\pi^0$ is depicted in part of Fig. 1 (a). According to the feynmann diagrams shown in Fig. 1, the scattering amplitudes \mathcal{M}_{J^P} with a exchanged $N^*(J^P)$ (including N) are given by:

$$\mathcal{M}_{\frac{1}{2}^+} = \frac{g_{\pi NP_{11}}}{2m_N} g_{VNP_{11}} \epsilon^\mu(p_1) \bar{u}(p_2) \left[\gamma_\mu \frac{-\not{p}_t + m_{N^*}}{t - m_{N^*}^2} \gamma_5 (i\not{p}_4) F(t) + \gamma_5 (i\not{p}_4) \frac{\not{p}_u + m_{N^*}}{u - m_{N^*}^2} \gamma_\mu F(u) \right] v(p_3), \quad (23)$$

$$\mathcal{M}_{\frac{1}{2}^-} = g_{\pi NS_{11}} g_{VNS_{11}} \epsilon^\mu(p_1) \bar{u}(p_2) \left[\gamma_5 \gamma_\mu \frac{-\not{p}_t + m_{N^*}}{t - m_{N^*}^2} F(t) + \frac{\not{p}_u + m_{N^*}}{u - m_{N^*}^2} \gamma_5 \gamma_\mu F(u) \right] v(p_3), \quad (24)$$

$$\mathcal{M}_{\frac{3}{2}^+} = \frac{g_{\pi NP_{13}}}{m_N} i g_{VNP_{13}} \epsilon^\mu(p_1) \bar{u}(p_2) \left[\gamma_5 \frac{-\not{p}_t + m_{N^*}}{t - m_{N^*}^2} G_{\mu\nu}(-p_t) (i p_4^\nu) F(t) + (i p_4^\nu) \frac{\not{p}_u + m_{N^*}}{u - m_{N^*}^2} G_{\nu\mu}(p_u) \gamma_5 F(u) \right] v(p_3), \quad (25)$$

$$\mathcal{M}_{\frac{3}{2}^-} = \frac{g_{\pi ND_{13}}}{m_N^2} g_{VND_{13}} \epsilon^\mu(p_1) \bar{u}(p_2) \left[\frac{-\not{p}_t + m_{N^*}}{t - m_{N^*}^2} G_{\mu\nu}(-p_t) \gamma_5 (i\not{p}_4) (i p_4^\nu) F(t) + \gamma_5 (i\not{p}_4) (i p_4^\nu) \frac{\not{p}_u + m_{N^*}}{u - m_{N^*}^2} G_{\nu\mu}(p_u) F(u) \right] v(p_3) \quad (26)$$

with $t = p_t^2 = (p_1 - p_2)^2$ and $u = p_u^2 = (p_1 - p_3)^2$, and $G_{\mu\nu}$ is

$$G_{\mu\nu}(p) = (-g_{\mu\nu} + \frac{1}{3} \gamma_\mu \gamma_\nu + \frac{1}{3m_{N^*}} (\gamma_\mu p_\nu - \gamma_\nu p_\mu) + \frac{2}{3} \frac{p_\mu p_\nu}{m_{N^*}^2}). \quad (27)$$

For $\mathcal{M}_{N^*(N)}$ in Eq. (18), just drop the polarization vector $\epsilon^\mu(p_1)$.

For $p\bar{p} \rightarrow \pi^0 \psi(3770)$ reaction, the scattering amplitudes can be easily obtained just applying the substitution to Eqs. (23-26):

$$p_1 \rightarrow -p_3, \quad p_2 \rightarrow -p_2, \quad p_3 \rightarrow -p_1, \quad p_t \rightarrow -p_u. \quad (28)$$

The amplitude of the process $\psi(3686) \rightarrow p\bar{p}\pi^0$ and $p\bar{p} \rightarrow \pi^0 \psi(3686)$ is exactly the same.

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